

with the Taylor expansion where the Maclaurin series were cut off, thus bypassing the equilibrium and compatibility conditions at B . It is obvious that M_A and V_A can be obtained by equilibrium considerations.

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Nonequilibrium Electrical Conductivity Measurements in Argon and Helium Seeded Plasmas

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IN a previous paper,¹ the authors presented experimental values of electrical conductivity measured in a plasma composed of argon gas seeded with potassium vapor. The measurements were made at atmospheric pressure with a neutral gas temperature of $2000^\circ \pm 100^\circ \text{K}$ and with a number of values of seed concentration in the range 0.2 to 0.8 mole %. The effect of nonequilibrium heating of the electron gas-excited potassium system was investigated for a range of current densities between 0.8 and 80 amp/cm². These data were in good agreement with values of the conductivity calculated by a scheme, outlined in Ref. 1, which included the effects of energy loss from the system, composed of the electron gas and the electronically excited states of potassium due to radiation from the excited potassium atoms. In addition, the pulsed technique used to measure the conductivity in response to a step function application of the electric field made possible the determination of the relaxation times for the ionization process.

The purpose of the present note is to present more conductivity measurements for the argon-potassium system, which extend the range of neutral gas temperatures and seed

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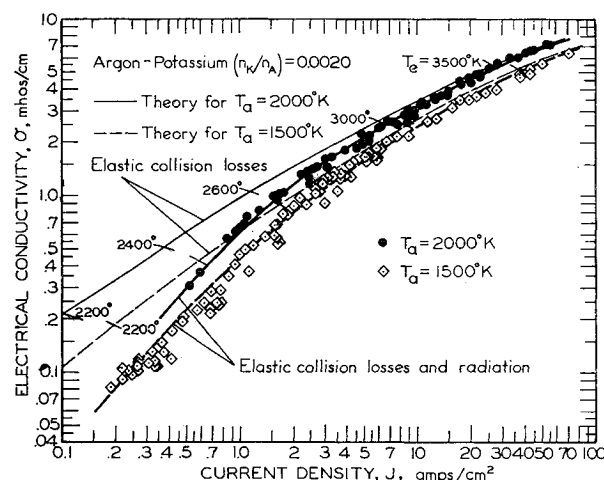


Fig. 1 Dependence of steady-state conductivity on current density.

concentrations investigated. In addition, measurements of conductivity and relaxation times for the helium-potassium system have been made. The apparatus and experimental techniques were identical, except for a few refinements, to those described in Ref. 1. The calculation scheme was also the same except for re-estimation of radiation losses for the helium-potassium system and appropriate modification of Eqs. (7-12) to include atomic species of differing masses.

Consider first the conductivity measurements made in the argon-potassium system. Experimental and calculated values of conductivity are presented as a function of the current density in Figs. 1-3. Calculated values of the electron temperature T_e are also indicated on the curves. The conductivity measurements presented here are values obtained with neutral gas temperatures T_g of $1500^\circ \pm 30^\circ \text{K}$ and $2000^\circ \pm 100^\circ \text{K}$; in all cases, the pressure is 1 atm. In general, the data shown here agree well with the calculated values. In addition to these data, similar results have been obtained for temperatures of 1750° and 1250°K , and measurements at 2000°K were extended to include a mole fraction of 0.1%. The agreement with calculated values and scatter in the data is similar to that for the data shown here.

Note that radiation is the dominant loss mechanism in the low current range, and that measured and calculated values are still in good agreement down to the lowest current densities shown in the figures, about 0.2 and 0.4 amp/cm² for the 1500° and 2000°K gas temperatures, respectively. Hence, there is no reason to suspect that the two-temperature model used in

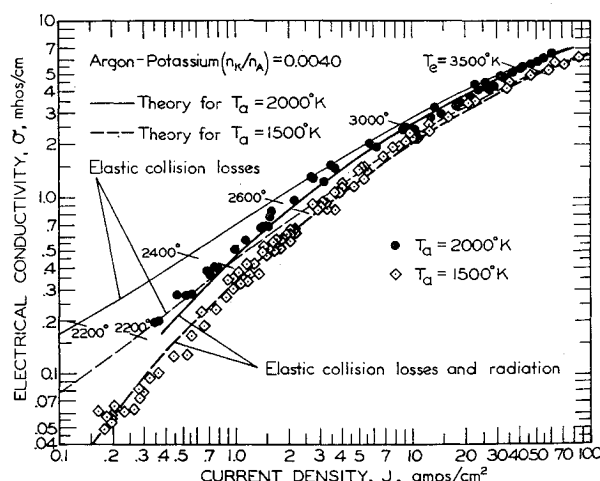


Fig. 2 Dependence of steady-state conductivity on current density.

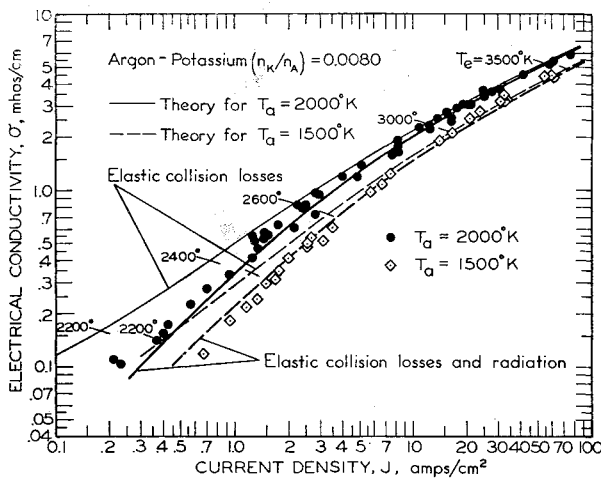


Fig. 3 Dependence of steady-state conductivity on current density.

the calculation fails in this low current region because of the absence of a Maxwellian distribution of the electrons, as has been suggested by Kerrebrock.²

However, for current densities between the limiting values given in the last paragraph and the lowest current density at which measurements were made, about 0.08 amp/cm², the conductivity values are almost constant and thus lie above the calculated values. Because of problems encountered with probe measurements when the current density was very small, the authors could not be sure of the validity of these very low current-density data, and hence did not include these results. Certainly it would not be surprising if the calculations were not valid below these limits, since many of the assumptions made in the calculations must fail as the current density approaches zero. At these limiting conditions, calculated values of electron density are about 7×10^{12} and 2×10^{13} per cm³, respectively.

The measurements made in the helium-potassium system at $2000^\circ \pm 100^\circ \text{K}$ and 0.32 mole % potassium are presented in Fig. 4, where they are compared with calculated values for both the helium- and argon-potassium systems. In making the calculations, a mean value of $5.1 \times 10^{-16} \text{ cm}^2$ of the momentum-transfer cross section for helium was used in agreement with the work of Gould and Brown and that of Normand as discussed in Ref. 3. Ramsauer and Kollath obtained total cross section values about 15% higher than those of Normand, as shown in Ref. 4. These values would give an average momentum-transfer cross section of about 6×10^{-16}

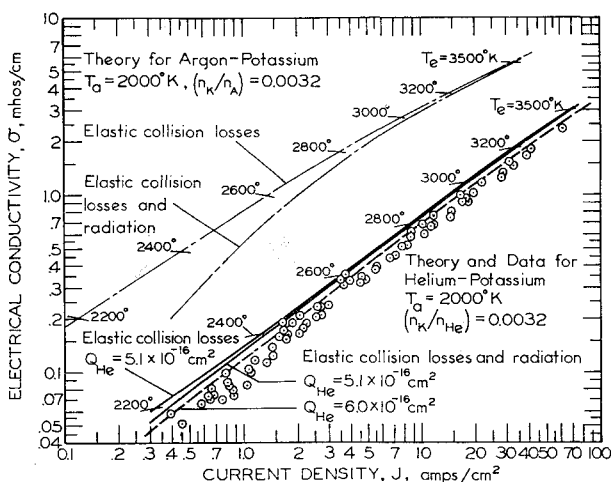


Fig. 4 Dependence of steady-state conductivity on current density.

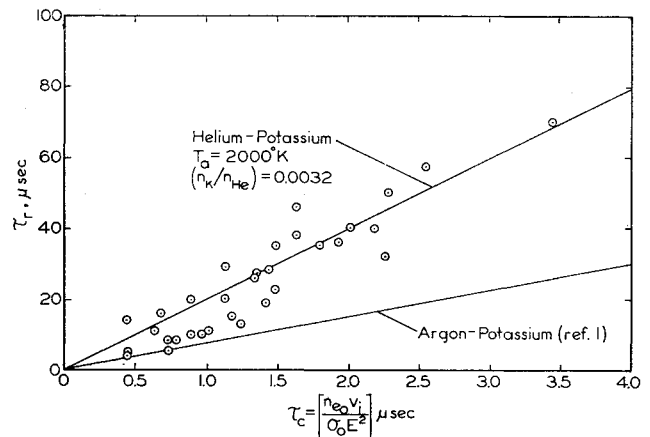


Fig. 5 Variation of relaxation time with characteristic time.

cm². The corresponding theoretical curve (dashed) is also shown with the data of Fig. 4. The agreement between both calculated curves and the measured values is satisfactory down to the 0.4 amp/cm² limit shown here. Again, as in the argon case, conductivity values for current densities between 0.08 amp/cm² and this limit remained nearly constant and are also of questionable validity at present.

The radiation correction for the helium-potassium system is much smaller than that for the argon-potassium system. For example, at 1 amp/cm², the radiation correction produces a reduction in conductivity in the helium- and argon-potassium systems of about 5% and over 30%, respectively. This difference is produced by the increase in elastic energy transfer between electrons and neutrals resulting from the greatly increased momentum-transfer cross section and the reduced atomic weight of helium as compared with argon. The large decrease in conductivity which occurs when helium is substituted for argon, e.g., Fig. 4, is a result of the same changes.

Measured values of relaxation times for the helium-potassium system are shown in Fig. 5 as a function of a characteristic time given by $(n_{eo} V_i) / (\sigma_o E^2)$. Here, E is the applied field strength, n_{eo} and σ_o are the electron density and conductivity evaluated at the gas temperature, and V_i is the ionization potential for potassium. The data lie roughly along a straight line with a slope of about 20:1. Since the residence time for the gas between the electrodes is about 150 μsec , data obtained for relaxation times greater than about 50 μsec are suspect. The slope of this curve is about 3 times larger than that for the argon-potassium system. This difference is probably due to the increased importance of elastic energy exchange in the helium-potassium system as compared with that in the argon-potassium system.

The good agreement between experimental and calculated values of conductivity demonstrated here for the argon-potassium and helium-potassium systems indicates that the computational scheme discussed in Ref. 1 is reasonably accurate over a wide range of parameters. This range includes conditions at which scattering of electrons by neutral atoms and ions are equally important.

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